Équations différentielles I

Quelques solutions du TD2

Solution 3.

a) $x(t) = \lambda_1 e^t + \lambda_2 e^{-3t} + \frac{3t-1}{9}$, $\lambda_1, \lambda_2 \in \mathbb{R}$.
$x(0) = 0, x'(0) = 1 \iff \lambda_1 = \frac{1}{4}, \lambda_2 = \frac{-5}{36}$.

b) $x(t) = \lambda_1 e^t + \lambda_2 e^{-3t} + \frac{1}{5}e^{2t}$, $\lambda_1, \lambda_2 \in \mathbb{R}$.
$x(0) = 0, x'(0) = 1 \iff \lambda_1 = 0, \lambda_2 = \frac{-1}{5}$.

c) $x(t) = \lambda_1 e^t + \lambda_2 e^{-3t} + \frac{1}{90}(-10 + 18e^{2t} + 30t - 18\cos(t) + 9\sin(t))$, $\lambda_1, \lambda_2 \in \mathbb{R}$.
$x(0) = 0, x'(0) = 1 \iff \lambda_1 = \frac{-1}{72}, \lambda_2 = \frac{1}{8}$.

d) $x(t) = \lambda_1 e^{3t} + \lambda_2 t e^{3t} + \frac{1}{12}(4 + 3e^t)$, $\lambda_1, \lambda_2 \in \mathbb{R}$.
$x(0) = 0, x'(0) = 1 \iff \lambda_1 = \frac{-7}{12}, \lambda_2 = \frac{5}{2}$.

e) $x(t) = \lambda_1 e^{3t} + \lambda_2 + \frac{1}{10}e^{-2t}$, $\lambda_1, \lambda_2 \in \mathbb{R}$.
$x(0) = 0, x'(0) = 1 \iff \lambda_1 = \frac{2}{5}, \lambda_2 = \frac{-1}{2}$.

f) $x(t) = \lambda_1 e^{3t} + \lambda_2 - \frac{1}{27}(29t + 3t^2 + 3t^3)$, $\lambda_1, \lambda_2 \in \mathbb{R}$.
$x(0) = 0, x'(0) = 1 \iff \lambda_1 = \frac{56}{81}, \lambda_2 = \frac{-56}{81}$.

g) Par cas :
- Si $\omega \neq \pm 1$: $x(t) = \lambda_1 \cos(t) + \lambda_2 \sin(t) - \frac{1}{\omega^2 - 1}\sin(\omega t)$, $\lambda_1, \lambda_2 \in \mathbb{R}$.
$x(0) = 0, x'(0) = 1 \iff \lambda_1 = 0, \lambda_2 = \frac{\omega^2 + \omega + 1}{\omega^2 - 1}$.
- Si $\omega = 1$: $x(t) = \lambda_1 \cos(t) + \lambda_2 \sin(t) - \frac{t}{2}\cos(t)$, $\lambda_1, \lambda_2 \in \mathbb{R}$.
$x(0) = 0, x'(0) = 1 \iff \lambda_1 = 0, \lambda_2 = \frac{3}{2}$.
- Si $\omega = -1$: $x(t) = \lambda_1 \cos(t) + \lambda_2 \sin(t) + \frac{t}{2}\cos(t)$, $\lambda_1, \lambda_2 \in \mathbb{R}$.
$x(0) = 0, x'(0) = 1 \iff \lambda_1 = 0, \lambda_2 = \frac{1}{2}$.

h) $x(t) = \lambda_1 \cos(t) + \lambda_2 \sin(t)t - \frac{1}{3}\sin(2t)$, $\lambda_1, \lambda_2 \in \mathbb{R}$.
$x(0) = 0, x'(0) = 1 \iff \lambda_1 = \frac{2}{3}, \lambda_2 = 0$.

Solution 5.

a) $x(t) = \frac{e^{-6t}}{49} - \frac{e^t}{49} + \frac{e^t t}{7}$.

b) $x(t) = -10e^{-6t} + 12e^t + 105e^{2t} - 72e^{2t}t + 32e^{2t}t^2$.

c) $x(t) = -\frac{13}{105}e^{-6t} + \frac{11e^t}{70} - \frac{1}{30}\cos(3t) + \frac{1}{30}\sin(3t)$.

d) $x(t) = -\frac{4}{5}e^{-2t} + \frac{7e^{-t}}{4} + \frac{e^{3t}}{20}$.

e) $x(t) = \frac{1}{2}e^{4t}\left(2t + t^2\right)$.

f) $x(t) = 3 + \cos(3t) + 2\sin(3t)$.

g) $x(t) = -\frac{5}{9} + \frac{17e^{-3t}}{144} + \frac{7e^t}{16} - \frac{t}{3} + \frac{e^t t}{4}$.

h) $x(t) = -2 + e^{1+t} - t^2$.

i) $y(t) = t$.

j) $y(t) = t + e^{-t/2}\cos\left(\frac{\sqrt{3}t}{2}\right) - \frac{e^{-t/2}\sin\left(\frac{\sqrt{3}t}{2}\right)}{\sqrt{3}}$.

k) $y(t) = \frac{1}{4}(t - (-4 + t)\cos(2t))$.

l) $y(t) = \frac{1}{12} + \frac{e^{-4t}}{60} - \frac{e^t}{10} - \frac{e^{2t}}{6} + \frac{7e^{3t}}{6}$.

m) $y(t) = 7 - \frac{27e^{-t}}{4} - \frac{e^t}{4} - 4t - \frac{e^{-t}t}{2} + t^2$.

n) $y(t) = -2 + \frac{3e^{-t}}{4} + \frac{5e^t}{4} - \frac{e^{-t}t}{2}$.

o) $y(t) = \frac{7e^{-t}}{12} + \frac{3e^t}{4} - \frac{e^{2t}}{3} - \frac{\sin(t)}{2}$.

p) $y(t) = \frac{1}{20}(-\cos(2t) + 6\cos(t)(-1+\sin(t)) - 2\sin(t))$.